

SOME CONSTRUCTIONS FOR K-RECONSTRUCTION

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1. Introduction.

The purpose of this article is to exhibit some constructions associated with the idea of k -reconstruction, as described in [2], and thereby to deduce some bounds for the function $v_R(k)$ defined in [2]. We also include a list of all known non-3-reconstructible unordered triples of graphs.

We begin by stating the appropriate definitions. Undefined symbols or terms may be found in [1] and/or [2].

DEFINITION: The *deck*, $D[G]$, of a graph G is the multiset consisting of the collection of vertex-deleted subgraphs of G . Each vertex-deleted subgraph is called a *card* of G .

Example:



DEFINITION: Let G_1, G_2, \dots, G_k be k graphs such that $v(G_1) = v(G_2) = \dots = v(G_k)$. The *shuffled k -deck*, $D[G_1, G_2, \dots, G_k]$, of the unordered k -tuple of graphs $[G_1, G_2, \dots, G_k]$ is the multiset formed by shuffling together the individual decks $D[G_1], D[G_2], \dots, D[G_k]$.

DEFINITION: An unordered k -tuple of graphs, $[G_1, G_2, \dots, G_k]$ is k -reconstructible if there do not exist graphs H_1, H_2, \dots, H_k such that $D[G_1, G_2, \dots, G_k] = D[H_1, H_2, \dots, H_k]$, but $[G_1, G_2, \dots, G_k] \neq [H_1, H_2, \dots, H_k]$.

If Γ is a class of graphs (e.g., trees, disconnected graphs, etc.) such that all unordered k -tuples of graphs from Γ are k -reconstructible, we say that Γ is k -reconstructible.

The following two theorems were proved in [2].

THEOREM 1. *Let G_1 and G_2 be disconnected graphs such that $v(G_1) = v(G_2) \geq 5$ and $\omega(G_1) + \omega(G_2) \geq 6$. Then $\{G_1, G_2\}$ is 2-reconstructible.*

THEOREM 2. *The class of disconnected graphs with at least five vertices is 2-reconstructible if and only if the reconstruction conjecture is true.*

These two theorems suggest that perhaps all graphs with at least five vertices are 2-reconstructible if the reconstruction conjecture is true. Indeed, it was found (see [2]) that although there are several non-2-reconstructible pairs of graphs on two, three and four vertices, there are none on five or six vertices. It proved too difficult to examine the 1044 graphs on seven vertices.

It was suggested in [2] that there exists a function $v_R(k)$ such that all graphs with at least $v_R(k)$ vertices are k -reconstructible. In particular, $v_R(1) = 3$ corresponds to the reconstruction conjecture, and $v_R(2) = 5$ is strongly suggested by [2]. We show below that $v_R(k) \geq 3 + 2 \lceil \log k \rceil$, and we suspect that in fact, $v_R(k) = 3 + 2 \lceil \log k \rceil$, where the upper brackets, $\lceil x \rceil$, represent the least integer greater than or equal to x . Here, and in the sequel, all logarithms are taken to base two.

k	$3+2 \lceil \log k \rceil$	$3+2 \lceil \log k \rceil$
1	3	3
2	5	5
3	5	7
4	7	7
5	7	9
6	7	9
7	7	9
8	9	9

Table 1.

2. Non-K-Reconstructible Families.

Let $H_{m,n}$ denote the graph on $2m$ vertices formed by taking the disjoint union of n edges and $2(m-n)$ isolated points.

Example:

$$H_{3,3} = \begin{array}{c} \bullet \text{---} \bullet \\ \bullet \text{---} \bullet \\ \bullet \text{---} \bullet \end{array}$$

$$H_{3,1} = \begin{array}{c} \bullet \text{---} \bullet \\ \vdots \\ \vdots \end{array}$$

THEOREM 3. Let $m \geq 1$. Let $k = 2^{m-1}$. Let G represent the unordered k -tuple of graphs containing

$\binom{m}{0}$ copies of $H_{m,m}$,

$\binom{m}{2}$ copies of $H_{m,m-2}$,

$\binom{m}{4}$ copies of $H_{m,m-4}$,

...,

$\binom{m}{2\lceil m/2 \rceil}$ copies of $H_{m,m-2\lceil m/2 \rceil}$.

Let H represent the unordered k -tuple of graphs containing:

$$\begin{aligned} & \binom{m}{1} \text{ copies of } H_{m,m-1}, \\ & \binom{m}{3} \text{ copies of } H_{m,m-3}, \\ & \dots, \\ & \binom{m}{2[(m+1)/2]-1} \text{ copies of } H_{m,m-2[(m+1)/2]+1}. \end{aligned}$$

Then $D[G] = D[H]$, but $G \neq H$.

Proof: The proof is best illustrated by an example. Let $m = 3$. Then the theorem states that

$$D \left[\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \end{array} \right] = D \left[\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \end{array} \right].$$

COROLLARY 4. $v_R(k) \geq 3 + 2 \lfloor \log k \rfloor$.

Proof: The construction of Theorem 3 provides a non- 2^{m-1} -reconstructible set of graphs on $2m$ vertices. Thus $v_R(2^{m-1}) \geq 2m+1$. If $k = 2^{m-1}$, this implies that $v_R(k) \geq 3 + 2 \lfloor \log k \rfloor$, where we have used the fact that $v_R(k+1) \geq v_R(k)$ (see [2]).

Now let $X_{m,n}$ denote the graph on $2m$ vertices formed by choosing a complete m -graph, K_m , and appending a vertex of degree one to n of its vertices, and then adding $m-n$ isolated points.

Example:

$$\begin{aligned} X_{4,4} &= \begin{array}{c} \bullet \quad \bullet \\ \diagup \quad \diagdown \\ \square \\ \diagdown \quad \diagup \\ \bullet \quad \bullet \end{array} \\ X_{4,2} &= \begin{array}{c} \bullet \quad \bullet \\ \diagup \quad \diagdown \\ \square \\ \diagdown \quad \diagup \\ \bullet \quad \bullet \\ \bullet \quad \bullet \end{array} \end{aligned}$$

THEOREM 5. Let $m \geq 1$. Let $k = 2^{m-1}$. Let G represent the unordered k -tuple of graphs containing:

$\binom{m}{0}$ copies of $X_{m,m}$,

$\binom{m}{2}$ copies of $X_{m,m-2}$,

....,

$\binom{m}{2[m/2]}$ copies of $X_{m,m-2[m/2]}$.

Let H represent the unordered k -tuple of graphs containing:

$\binom{m}{1}$ copies of $X_{m,m-1}$,

$\binom{m}{3}$ copies of $X_{m,m-3}$,

....,

$\binom{m}{2[(m+1)/2]-1}$ copies of $X_{m,m-2[(m+1)/2]+1}$.

Then $D[G] = D[H]$, but $G \neq H$.

Proof: This construction is very similar to that of Theorem 3. If $m = 3$, say, the theorem states that

$$D \left[\begin{array}{c} \triangle \\ \cdot \end{array}, \begin{array}{c} \triangle \\ \cdot \end{array}, \begin{array}{c} \triangle \\ \cdot \end{array}, \begin{array}{c} \triangle \\ \cdot \end{array} \right] = D \left[\begin{array}{c} \triangle \\ \cdot \end{array}, \begin{array}{c} \triangle \\ \cdot \end{array}, \begin{array}{c} \triangle \\ \cdot \end{array}, \begin{array}{c} \triangle \\ \cdot \end{array} \right] .$$

This is easily seen to be true.

Note that like Theorem 3, Theorem 5 also constructs a non- 2^{m-1} -reconstructible set of graphs on $2m$ vertices.

Referring to the preceeding construction, note that there are $\binom{m}{n}$ graphs with n isolated points. Since

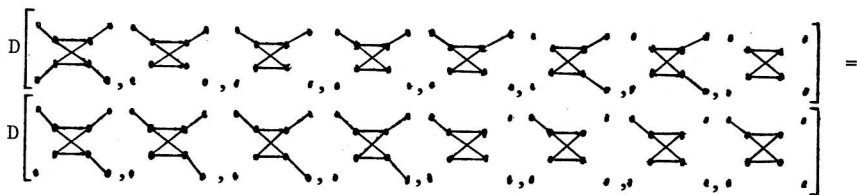
$$(m-n)\binom{m}{n} = (n+1)\binom{m}{n+1},$$

we are assured that the shuffled k -decks $D[G]$ and $D[H]$ are indeed equal. We may extend Theorem 5 by extending the above binomial identity to a multinomial identity:

$$(n_1+1)\binom{m}{n_1+1, n_2, n_3} = (n_2+1)\binom{m}{n_1, n_2+1, n_3} = (n_3+1)\binom{m}{n_1, n_2, n_3+1}.$$

This is done by replacing the complete graph K_m used in forming $X_{m,n}$ by a complete multipartite graph $K_{n_1, n_2, \dots}$ where $n_1+n_2+\dots = m$.

Example: Replace K_4 with $K_{2,2}$.



Naturally, we need not have $n_1 = n_2 = \dots$. It is clear that these multipartite families all produce non- 2^{m-1} -reconstructible sets of graphs on $2m$ vertices.

THEOREM 6. Let $D[G_1, G_2, \dots, G_{k_1}] = D[H_1, H_2, \dots, H_{k_1}]$ and $D[G'_1, G'_2, \dots, G'_{k_2}] = D[H'_1, H'_2, \dots, H'_{k_2}]$ be examples of non- k_1 -reconstructible and non- k_2 -reconstructible sets of graphs on v_1 and v_2 vertices,

respectively. Then we may multiply these examples together to produce the following non- $2k_1 k_2$ -reconstructible set of graphs on $v_1 + v_2$ vertices. The plus sign used below indicates the operation of disjoint union of graphs.

$$\begin{aligned}
& D[G_1 + G'_1, G_1 + G'_2, \dots, G_1 + G'_{k_2}, \\
& \quad G_2 + G'_1, G_2 + G'_2, \dots, G_2 + G'_{k_2}, \\
& \quad \dots, \\
& \quad G_{k_1} + G'_1, G_{k_1} + G'_2, \dots, G_{k_1} + G'_{k_2}, \\
& \quad H_1 + H'_1, H_1 + H'_2, \dots, H_1 + H'_{k_2}, \\
& \quad \dots, \\
& \quad H_{k_1} + H'_1, H_{k_1} + H'_2, \dots, H_{k_1} + H'_{k_2}] = \\
& D[G_1 + H'_1, G_1 + H'_2, \dots, G_1 + H'_{k_2}, \\
& \quad \dots, \\
& \quad G_{k_1} + H'_1, G_{k_1} + H'_2, \dots, G_{k_1} + H'_{k_2}, \\
& \quad H_1 + G'_1, H_1 + G'_2, \dots, H_1 + G'_{k_2}, \\
& \quad \dots, \\
& \quad H_{k_1} + G'_1, H_{k_1} + G'_2, \dots, H_{k_1} + G'_{k_2}] .
\end{aligned}$$

Proof: Each G_1 occurs once with each of $G'_1, G'_2, \dots, G'_{k_2}$ on the LHS and once with each of $H'_1, H'_2, \dots, H'_{k_2}$ on the RHS. Note that $D[G'_1, G'_2, \dots, G'_{k_2}] = D[H'_1, H'_2, \dots, H'_{k_2}]$. Similarly each G'_1 occurs once with each of G_1, G_2, \dots, G_{k_1} on the LHS, and once with each of H_1, H_2, \dots, H_{k_1} on the RHS. Note that $D[G_1, G_2, \dots, G_{k_1}] = D[H_1, H_2, \dots, H_{k_1}]$.

Similar observations apply to H_1 and H'_1 .

COROLLARY 7. $v_R(2k_1k_2) \geq v_R(k_1) + v_R(k_2) - 1$.

Proof: By multiplying a non- k_1 -reconstructible set of graphs on $v_R(k_1) - 1$ vertices with a non- k_2 -reconstructible set on $v_R(k_2) - 1$ vertices, we obtain a non- $2k_1k_2$ -reconstructible set on $v_R(k_1) + v_R(k_2) - 2$ vertices. Thus $v_R(2k_1k_2) \geq v_R(k_1) + v_R(k_2) - 1$.

COROLLARY 8. $v_R(2k^2) \geq 2v_R(k) - 1$.

COROLLARY 9. $v_R(2k) \geq v_R(k) + 2$.

Proof: Set $k_1 = k$ and $k_2 = 1$ in Corollary 7.

Note that by iterating Corollary 9 beginning with $v_R(1) = 3$, we obtain Corollary 4, namely $v_R(k) \geq 3 + 2 \lceil \log k \rceil$. We can improve this a bit, as follows.

COROLLARY 10. $v_R(k) \geq \max\{7+6\lceil(\log k/3)/\log 6\rceil, 3+6\lceil\log k/\log 6\rceil, 5+6\lceil(\log k/2)/\log 6\rceil\}$.

Proof: The example

$$D[\text{cyclohexagon}, \text{triangle}, \text{square}] = D[\text{cyclohexagon}, \text{square with 3 internal edges}, \text{square with 4 internal edges}]$$

demonstrates that $v_R(3) \geq 7$. Substituting this into Corollary 7 yields

$$v_R(6k) \geq v_R(k) + 6.$$

If we now iterate this, beginning with $v_R(3) \geq 7$, we obtain

$$v_R(3 \cdot 6^n) \geq 6n + 7,$$

which holds for $n \geq 0$. We now apply Corollary 9 twice to obtain

$$v_R(6^{n+1}) \geq 6n+9$$

$$v_R(2 \cdot 6^{n+1}) \geq 6n+11.$$

We now invert these three inequalities to obtain the stated result.

Note that in Theorem 3, if $m = 1$ the construction gives $D[K_2] = D[2K_1]$. We may thus consider the non-reconstructible unordered k -tuples of Theorem 3 as powers of the equality $D[K_2] = D[2K_1]$, where the powers are formed through the multiplication of Theorem 6.

3. Computer Generated K -Tuples.

A complete list of known non-2-reconstructible unordered pairs of graphs appears in [2]. In the following, we have used the computer to find all non-3-reconstructible unordered triples of graphs on 2,3,4,5 and 6 vertices. The following points should be kept in mind.

- 1) If $D[G_1, G_2, G_3] = D[H_1, H_2, H_3]$, then $D[\bar{G}_1, \bar{G}_2, \bar{G}_3] = D[\bar{H}_1, \bar{H}_2, \bar{H}_3]$, where the bar denotes the complement of a graph. Unless such an equality is self-complementary, we do not include the complement. Self-complementary examples are marked by an asterisk.
- 2) We do not include examples of the form $D[G, G_1, G_2] = D[G, H_1, H_2]$, as

these all may be constructed from the non-2-reconstructible pairs

$$D[G_1, G_2] = D[H_1, H_2].$$

$$v = 2.$$

$$* D[\dashv, \dashv, \dashv] = D[\cdot, \cdot, \cdot]$$

$$v = 3.$$

$$D[\ddot{\cdot}, \ddot{\cdot}, \Delta] = D[\dot{\cdot}, \dot{\cdot}, \dot{\cdot}]$$

$$v = 4.$$

$$* D[\dashv, \triangleright, \triangleright] = D[\ulcorner, \ulcorner, \Box]$$

$$* D[\sqcap, \triangleright, \wedge] = D[\swarrow, \ulcorner, \square]$$

$$D[\sqcap, \dashv, \triangleright] = D[\swarrow, \ulcorner, \ulcorner]$$

$$D[\sqcap, \ulcorner, \Box] = D[\swarrow, \swarrow, \wedge]$$

$$D[\ddot{\cdot}, \swarrow, \sqcup] = D[\dashv, \dashv, \triangleright]$$

$$D[\ddot{\cdot}, \swarrow, \triangleright] = D[\dashv, \dashv, \Box]$$

$$D[\ddot{\cdot}, \ulcorner, \sqcup] = D[\dashv, \dashv, \wedge]$$

$$D[\square, \square, \dashv] = D[\wedge, \wedge, \sqcup]$$

$$D[\sqcap, \dashv, \wedge] = D[\ulcorner, \ulcorner, \ulcorner]$$

$$D[\ddot{\cdot}, \sqcup, \square] = D[\dashv, \wedge, \wedge]$$

$$D[\ddot{\cdot}, \sqcap, \sqcup] = D[\ulcorner, \ulcorner, \dashv]$$

$$D[\sqcap, \sqcup, \Box] = D[\swarrow, \swarrow, \square] = D[\sqcap, \triangleright, \triangleright]$$

$$D[\sqcap, \wedge, \Box] = D[\ulcorner, \triangleright, \triangleright] = D[\wedge, \triangleright, \swarrow] = D[\Box, \sqcup, \ulcorner]$$

$$D[\sqcap, \ulcorner, \wedge] = D[\dashv, \sqcup, \sqcup] = D[\sqcap, \dashv, \square] = D[\ulcorner, \ulcorner, \sqcup]$$

$$D[\ddot{\cdot}, \sqcap, \square] = D[\dashv, \ulcorner, \wedge] = D[\ddot{\cdot}, \sqcup, \sqcup] = D[\dashv, \dashv, \square]$$

$$* \quad D[\text{diagram 1}, \text{diagram 2}, \text{diagram 3}] = D[\text{diagram 4}, \text{diagram 5}, \text{diagram 6}]$$

* $D[\triangleleft, \square, \triangle] = D[\triangleright, \triangle, \square]$

$$* \quad D[\text{triangle with dot at top-left}, \text{triangle with dot at top-right}, \text{triangle with dot at bottom}] = D[\text{bowtie}, \text{square with dot at center}, \text{triangle with dot at top}]$$

$$D[\text{---}, \text{---}, \text{---}] = D[\text{---}, \text{---}, \text{---}]$$

$$D[\text{diagram 1}, \text{diagram 2}, \text{diagram 3}] = D[\text{diagram 4}, \text{diagram 5}, \text{diagram 6}]$$

$$D[\text{trivalent vertex}, \text{triangle}, \text{triangle}] = D[\text{trivalent vertex}, \text{trivalent vertex}, \text{triangle}]$$

$$D[\overline{\cdot}, \triangleright, \sqcup] = D[\wedge, \vee, \triangleright]$$

$$D[\text{---}, \text{---}, \text{---}] = D[\text{---}, \text{---}, \text{---}]$$

$$D[\triangle, \triangle, \triangle] = D[\triangle, \triangle, \triangle]$$

$$D[\triangle, \text{Y}, \text{X}] = D[\text{A}, \text{B}, \text{C}]$$

$$D[\triangle, \frown, \bigtriangleup] = D[\sqcup, \triangleright, \triangleright\!\!\triangleright]$$

$$D[\triangle, \square, \triangle] = D[\wedge, \text{---}\triangle\text{---}, \text{---}\triangle\text{---}]$$

$$D[\triangle, \text{Y-shape}, \text{K4}] = D[\wedge, \text{X-shape}, \text{Square}]$$

$$D[\triangleleft, \triangle, \triangle] = D[\triangle, \triangleleft, \triangle]$$

$$D[\text{triangle}, \text{square}, \text{pentagon}] = D[\text{triangle}, \text{square}, \text{pentagon}]$$

$$D[\text{---}, \text{---}, \text{---}] = D[\text{---}, \text{---}, \text{---}]$$

$$D[\begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array}, \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array}, \begin{array}{c} \square \\ \square \\ \square \end{array}] = D[\begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array}, \begin{array}{c} \times \\ \times \\ \times \end{array}, \begin{array}{c} \square \\ \square \\ \square \end{array}]$$

$$D[\text{diagram 1}, \text{diagram 2}, \text{diagram 3}] = D[\text{diagram 4}, \text{diagram 5}, \text{diagram 6}]$$

$$D[\text{---}\cdot, \text{---}\cdot, \text{---}\cdot] = D[\text{---}\cdot, \text{---}\cdot, \text{---}\cdot]$$

$$D[\text{triangle}, \text{triangle}, \text{square}] = D[\text{triangle}, \text{triangle}, \text{triangle}]$$

$$D[\text{triangle}, \text{triangle}, \text{square}] = D[\text{square}, \text{cross}, \text{pentagon}]$$

$$D[\text{Y-shape}, \text{A-shape}, \text{square}] = D[\text{X-shape}, \text{dot}, \text{pentagon}]$$

$$D[\text{diagram 1}, \text{diagram 2}, \text{diagram 3}] = D[\text{diagram 4}, \text{diagram 5}, \text{diagram 6}]$$

$$\begin{aligned}
D[\text{III}, \text{I}, \langle \rangle] &= D[\text{I}, \text{I}, \text{I}] \\
D[\text{III}, \text{Y}, \langle \rangle] &= D[\text{I}, \text{I}, \text{Y}] \\
D[\text{III}, \text{X}, \langle \rangle] &= D[\text{I}, \text{I}, \text{X}] \\
D[\text{O}, \text{I}, \text{I}] &= D[\text{O}, \text{I}, \text{I}] \\
D[\text{O}, \text{I}, \text{O}] &= D[\text{O}, \text{O}, \text{O}] \\
D[\text{X}, \text{I}, \text{I}] &= D[\text{X}, \text{X}, \text{Y}] \\
D[\text{O}, \text{Y}, \text{I}] &= D[\text{I}, \text{O}, \text{O}] \\
D[\text{I}, \text{I}, \langle \rangle] &= D[\text{I}, \text{I}, \text{I}] \\
D[\text{I}, \text{Y}, \langle \rangle] &= D[\text{I}, \text{I}, \text{Y}] \\
D[\text{I}, \text{X}, \langle \rangle] &= D[\text{I}, \text{I}, \text{X}] \\
D[\text{I}, \text{X}, \text{O}] &= D[\text{Y}, \text{I}, \text{O}] \\
D[\text{I}, \text{I}, \text{O}] &= D[\text{I}, \text{I}, \text{I}] \\
D[\text{X}, \text{I}, \text{Y}] &= D[\text{Y}, \text{Y}, \text{Y}] \\
D[\langle \rangle, \text{Y}, \text{I}] &= D[\text{Y}, \text{Y}, \text{I}] \\
D[\langle \rangle, \text{O}, \text{O}] &= D[\text{I}, \text{O}, \text{Y}] \\
D[\langle \rangle, \text{O}, \text{O}] &= D[\text{I}, \text{Y}, \text{X}] \\
D[\text{I}, \text{Y}, \text{I}] &= D[\text{Y}, \text{I}, \text{I}] \\
D[\text{I}, \text{X}, \text{O}] &= D[\text{Y}, \text{I}, \text{X}] \\
D[\text{X}, \text{I}, \text{X}] &= D[\text{Y}, \text{X}, \text{X}] \\
D[\text{X}, \text{X}, \text{X}] &= D[\text{X}, \text{X}, \text{X}] \\
D[\text{Y}, \text{I}, \text{O}] &= D[\text{I}, \text{O}, \text{Y}] \\
D[\text{Y}, \text{Y}, \text{Y}] &= D[\text{Y}, \text{Y}, \text{Y}]
\end{aligned}$$

$$\begin{aligned}
D[\text{graph 1}, \text{graph 2}, \text{graph 3}] &= D[\text{graph 4}, \text{graph 5}, \text{graph 6}] \\
D[\text{graph 7}, \text{graph 8}, \text{graph 9}] &= D[\text{graph 10}, \text{graph 11}, \text{graph 12}] \\
D[\text{graph 13}, \text{graph 14}, \text{graph 15}] &= D[\text{graph 16}, \text{graph 17}, \text{graph 18}] \\
D[\text{graph 19}, \text{graph 20}, \text{graph 21}] &= D[\text{graph 22}, \text{graph 23}, \text{graph 24}] \\
D[\text{graph 25}, \text{graph 26}, \text{graph 27}] &= D[\text{graph 28}, \text{graph 29}, \text{graph 30}] \\
D[\text{graph 31}, \text{graph 32}, \text{graph 33}] &= D[\text{graph 34}, \text{graph 35}, \text{graph 36}] \\
D[\text{graph 37}, \text{graph 38}, \text{graph 39}] &= D[\text{graph 40}, \text{graph 41}, \text{graph 42}] \\
D[\text{graph 43}, \text{graph 44}, \text{graph 45}] &= D[\text{graph 46}, \text{graph 47}, \text{graph 48}] \\
D[\text{graph 49}, \text{graph 50}, \text{graph 51}] &= D[\text{graph 52}, \text{graph 53}, \text{graph 54}] \\
D[\text{graph 55}, \text{graph 56}, \text{graph 57}] &= D[\text{graph 58}, \text{graph 59}, \text{graph 60}] \\
D[\text{graph 61}, \text{graph 62}, \text{graph 63}] &= D[\text{graph 64}, \text{graph 65}, \text{graph 66}] \\
D[\text{graph 67}, \text{graph 68}, \text{graph 69}] &= D[\text{graph 70}, \text{graph 71}, \text{graph 72}] \\
D[\text{graph 73}, \text{graph 74}, \text{graph 75}] &= D[\text{graph 76}, \text{graph 77}, \text{graph 78}] \\
D[\text{graph 79}, \text{graph 80}, \text{graph 81}] &= D[\text{graph 82}, \text{graph 83}, \text{graph 84}] \\
D[\text{graph 85}, \text{graph 86}, \text{graph 87}] &= D[\text{graph 88}, \text{graph 89}, \text{graph 90}] \\
D[\text{graph 91}, \text{graph 92}, \text{graph 93}] &= D[\text{graph 94}, \text{graph 95}, \text{graph 96}] \\
D[\text{graph 97}, \text{graph 98}, \text{graph 99}] &= D[\text{graph 100}, \text{graph 101}, \text{graph 102}] \\
D[\text{graph 103}, \text{graph 104}, \text{graph 105}] &= D[\text{graph 106}, \text{graph 107}, \text{graph 108}] \\
D[\text{graph 109}, \text{graph 110}, \text{graph 111}] &= D[\text{graph 112}, \text{graph 113}, \text{graph 114}] \\
D[\text{graph 115}, \text{graph 116}, \text{graph 117}] &= D[\text{graph 118}, \text{graph 119}, \text{graph 120}]
\end{aligned}$$

In order to construct the above examples of non-reconstructible triples, a complete list of graphs with six or fewer vertices was necessary. The tables of graphs in [3] were used for this purpose.

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