Arc-connectivity of regular digraphs with two orbits *

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Abstract A strongly connected digraph D is said to be maximally arc connected if its arc-connectivity $\lambda(D)$ attains its minimum degree $\delta(D)$. For any vertex x of D, the set $\{x^g | g \in Aut(D)\}$ is called an orbit of $Aut(D)$. Liu and Meng [Fengxia Liu, Jixiang Meng, Edge-Connectivity of regular graphs with two orbits, Discrete Math. 308 (2008) 3711-3717] proved that the edge-connectivity of a k-regular connected graph with two orbits and $girth \geqslant 5$ attains its regular degree k. In the present paper, we prove the existence of k-regular m-arc-connected digraphs with two orbits for some given integer k and m. Furthermore, we prove that the k-regular connected digraphs with two orbits, satisfying $girth \geqslant k$ are maximally arc connected. Finally, we give an example to show that the girth bound k is best possible.

Keywords: Arc-connectivity ; Orbit

1 Terminology and introduction

We consider finite digraphs without loops and parallel arcs. Let $D = (V, E)$ be a strongly connected digraph, $S, T \subseteq V(D)$. Define $(S,T) = \{ (x,y) \in E(D) \,|\, x \in S, y \in T\}$. An *arc cut* of D is an arc set of the form $(S, V \backslash S)$, where $\emptyset \neq S \subsetneq V(D)$. The *arc-connectivity* $\lambda(D)$ is the minimum size of all arc cuts in D.

Let $D = (V, E)$ be a strongly connected digraph. If (u, v) is an arc of D, then we say u *dominates* v. The vertices which dominate a vertex v are

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its *in-neighbors*, those which are dominated by the vertex v are its *out-neighbors*. Let $F \subsetneq V$ be a nonempty set. We set $\omega^+(F) = (F, V \backslash F)$, and $\omega^-(F) = (V \backslash F, F)$. Usually, abbreviate $\omega^+(\{x\})$ and $\omega^-(\{x\})$ to $\omega^+(x)$ and $\omega^-(x)$, respectively. $d^+(x) = |\omega^+(x)|$ and $d^-(x) = |\omega^-(x)|$ are out-degree and in-degree of x, respectively. A subset F of vertices is called an *arc fragment*, if $|\omega^+(F)| = \lambda(D)$. An arc fragment of D with minimum cardinality is called a λ-*atom* of D. The digraph D is said to be a *balanced* digraph, if $d^+(u) = d^-(u)$ for every vertex u of D. It is not hard to see that in such a digraph, $|\omega^+(F)| = |\omega^-(F)|$ for every F, where $\emptyset \neq F \subsetneq V(D)$. Clearly, every regular digraph is balanced. For terminologies not given here, we refer [2] for reference.

Let H be a strongly connected balanced digraph with vertex set $\{v_1, v_2, \cdots, v_l\}$, $d^+(v_1) = d^+(v_2) = \cdots = d^+(v_m) = k-1$ and $d^+(v_{m+1}) = d^+(v_{m+2}) = \cdots = d^+(v_l) = k$. The digraph $D_2(H)$ is constructed by taking two copies of H, H_1, H_2, and adding arcs $\{(v_{i,j}, v_{i,3-j}) | \, v_{i,j} \in V(H_j), v_{i,3-j} \in V(H_{3-j}), 1 \leqslant i \leqslant m, j = 1, 2\}$ between H_1 and H_2. Clearly, $D_2(H)$ is a k-regular m-arc-connected digraph.

It is well known that when the underlying topology of an interconnection network is modeled by a (strongly) connected (di)graph D, the connectivity or edge(arc)-connectivity of D is an important measurement for fault tolerance of the network. In the design of network topology, (di)graphs of high symmetry are often used because they usually have many desirable properties. For instance, vertex transitive (di)graphs are maximally edge(arc) connected and edge transitive graphs are maximally (vertex) connected [4, 6, 12, 14]. Let $\lambda(D)$ be the edge(arc)-connectivity of D, and $\delta(D)$ be the minimum degree of D. So, a (di)graph D is said to be *maximally edge(arc) connected* if $\lambda(D) = \delta(D)$. Let U be a subgroup of the symmetric group over a set S. We say that U *acts transitively* on a subset T of S if for any $h, l \in T$, there exists a permutation $\varphi \in U$ with $\varphi(h) = l$. Denote by $Aut(D)$ the automorphism group of D. A (di)graph is said to be *vertex transitive*, if for any two vertices u and v of D, there is an automorphism $\phi \in Aut(D)$, such that $\phi(u) = v$. An undirected graph G is said to be *edge transitive*, if for any two edges e and f, there is an automorphism $\phi \in Aut(G)$, such that $\phi(e) = f$. Similarly, a digraph D is said to be *arc transitive*, if for any two arcs e' and f', there is an automorphism $\phi \in Aut(D)$, such that $\phi(e') = f'$. Investigations on connectivity and edge(arc)-connectivity of transitive (di)graphs were made by several authors, for example, by [5, 8, 9, 10, 11, 13, 15].

For any vertex x of D, the set $\{x^g | g \in Aut(D)\}$ is called an orbit of $Aut(D)$. Vertex transitive (di)graphs are (di)graphs with one orbit, and maximally edge(arc) connected [4]. It is natural to consider the relation between the edge(arc)-connectivity and the number of orbits. In paper [7] Liu and Meng proved that the edge-connectivity of a k-regular connected graph with two orbits and *girth* $\geqslant 5$ attains its regular degree k. In the present paper, we prove the existence of k-regular m-arc-connected digraphs with two orbits for some given integer k and m, and give an analogously sufficient condition for digraphs with two orbits to be maximally arc connected. Finally, we give an example to show that our result is best possible.

2 Main results

Hamidoune [4] proved the following:

Proposition 2.1. *Any two distinct λ-atoms are vertex disjoint.*

Let $D = (V, E)$ be a k-regular connected digraph with two orbits. In this paper, we use X_1 and X_2 to denote the two orbits of $Aut(D)$. Let A be a λ-*atom* of D. Set $A_1 = A \cap X_1$ and $A_2 = A \cap X_2$. Then $A = A_1 \cup A_2$.

Liu and Meng [7] proved the following in the case of undirected graphs. Actually, it is also true for directed graphs.

Proposition 2.2. *Let $D = (V, E)$ be a connected digraph with two orbits. Let $A = A_1 \cup A_2$ be a λ-atom of D. $Y = D[A]$ and $Y_i = D[A_i]$ for $i = 1, 2$. Then $Aut(Y)$ acts transitively on both A_1 and A_2, and $Aut(Y_i)$ acts transitively on A_i, for $i = 1, 2$.*

Lemma 2.3. *Let D be a k-regular connected digraph with two orbits ($k \geqslant 2$) and $\lambda(D) < k$. Use notation as the above. If $A = A_1 \cup A_2$ be a λ-atom of D, then $|A_i| \geqslant 2$ ($i = 1, 2$).*

Lemma 2.4. *Let D be a k-regular connected digraph with two orbits ($k \geqslant 2$) and $\lambda(D) < k$. Then $\lambda(D) \neq 1$.*

By Lemma 2.4, we have every 2-regular connected digraph with two orbits is maximally arc connected. By Lemma 2.3, we have if D is a $k(\geqslant 3)$-regular connected digraph with two orbits, $\lambda(D) < k$ and $A = A_1 \cup A_2$ is a λ-atom of D, then $A_1, A_2 \neq \emptyset$. Combining this with Proposition 2.1, we have if D is not maximally arc connected, then all the λ-atoms of D are isomorphic, and both $D[A_1]$ and $D[A_2]$ are vertex transitive.

Let D be a k-regular connected digraph with two orbits, and $A = A_1 \cup A_2$ be a λ-*atom* of D. By Proposition 2.2, we know that $Aut(Y)$ acts transitively on A_i, then the vertices in A_i have the same outdegree and the same indegree in Y, and $Aut(Y_i)$ acts transitively on A_i, then $D[A_i]$ is a regular digraph. Thus, we have non-negative integers k_1, k_2, k'_1, k'_2, r_1, r_2 such that for any $x \in A_i$, the digraph $D[A_i]$ is r_i regular, and $k_i = |(x, A_{3-i})|$, $k'_i = |(A_{3-i}, x)|$, $r_i = |(x, A_i)|$, $k_i, k'_i \leqslant |A_{3-i}|$, $r_i < |A_i|$,

$|(x, V\backslash A)| = k - k_i - r_i \geqslant 0$, $|(V\backslash A, x)| = k - k_i' - r_i \geqslant 0$, $i = 1, 2$. Clearly, $k_1|A_1| = k_2'|A_2|$, $k_1'|A_1| = k_2|A_2|$.

For two vertex-disjoint digraphs D_1 and D_2, the *join* $D = D_1 \vee D_2$ is obtained from $D_1 \cup D_2$ by joining every vertex of D_1 to every vertex of D_2, as well as joining every vertex of D_2 to every vertex of D_1. In other words, any two vertices $v_1 \in V(D_1)$ and $v_2 \in V(D_2)$ are linked by an undirected edge.

In paper [7], Liu and Meng proved the following:

Proposition 2.5. *(i)If k, m are even, $1 < m \leqslant k$, then there exists a k-regular m-edge-connected graph G with two orbits.*

(ii)If k is odd, $1 < m \leqslant k$, then there exists a k-regular m-edge-connected graph G with two orbits.

This proposition is clearly true in the case of digraphs. In fact, for any regular degree k, we have the following result:

Theorem 2.6. *For any given positive integer m and k, satisfying $1 < m \leqslant k$, then there exits a k-regular m-arc-connected digraph D with two orbits.*

Proof. By Proposition 2.5, it suffices to consider the existence of k-regular m-arc-connected digraph D with two orbits, when m is odd and k is even. We construct D as follows:

Since k is even and m is odd, we have $m \geqslant 3$, $k-2$ is even and $k-m$ is odd. Define $X_{k-2,k-m}$ to be the undirected graph with vertex set $V = \{0, 1, 2, \cdots, k-3\}$ and edge set $E = \{(i,j) \in V \times V \,|\, j - i \equiv p\,(mod(k-2))\,and\,1 \leqslant p \leqslant \frac{k-m-1}{2}\,or\,p = \frac{k-2}{2}\}$. Clearly, $X_{k-2,k-m}$ is a $(k-m)$-regular vertex transitive graph. Let C_m be the directed cycle of length m, and $H' = X_{k-2,k-m} \vee C_m$. Clearly, H' is a balanced digraph, and in H', the degree of vertex in $V(X_{k-2,k-m})$ is k, the degree of vertex in C_m is $k-1$. Let $D = D_2(H')$. Then, it is clear that D is a k-regular m-arc-connected digraph with two orbits. □

Before proceeding, we give the following Lemma:

Lemma 2.7. *Let D be a k-regular ($k \geqslant 3$) connected digraph with two orbits and $\lambda(D) < k$. Use notation as the above. If $A = A_1 \cup A_2$ be a λ-atom of D, then only the vertices in one orbit of A have out-neighbors (in-neighbors) in $V\backslash A$.*

Proof. By contradiction. Since $0 < \lambda = |\omega^+(A)| = |A_1|(k - k_1 - r_1) + |A_2|(k - k_2 - r_2) < k$. Suppose both the vertices in A_1 and A_2 dominate the vertices in $V\backslash A$. Then we have $|A_1| + |A_2| \leqslant \lambda < k$. We consider two cases:

Case 1. If $|A_1| + |A_2| = \lambda < k$, then $k - k_1 - r_1 = k - k_2 - r_2 = 1$, this implies that $k_1 + r_1 = k_2 + r_2 = k - 1$. Thus, $k - 1 = k_2 + r_2 \leqslant |A_1| + |A_2| - 1 = \lambda - 1 < k - 1$, a contradiction.

Case 2. If $|A_1|+|A_2| < \lambda < k$, then $k_i+r_i \leqslant |A_1|+|A_2|-1 \leqslant \lambda-2 \leqslant k-3$ $(i = 1, 2)$, that is $k-k_i-r_i \geqslant 3$. Hence, $k > \lambda = |A_1|(k-k_1-r_1)+|A_2|(k-k_2 - r_2) \geqslant 3(|A_1| + |A_2|)$, this implies that $|A_1| + |A_2| \leqslant \frac{k-1}{3}$, and thus $k_i+r_i \leqslant |A_1|+|A_2|-1 \leqslant \frac{k-1}{3}-1 = \frac{k-4}{3}$. Then $k-k_i-r_i \geqslant k-\frac{k-4}{3} = \frac{2k+4}{3}$, and so $(k - k_2 - r_2)|A_2| \geqslant (\frac{2k+4}{3})|A_2|$. Without loss of generality, assume that $|A_1| \leqslant |A_2|$. Then $k'_2 \leqslant |A_1| \leqslant \frac{1}{2}(|A_1| + |A_2|) \leqslant \frac{k-1}{6}$ and so $k'_2|A_2| \leqslant \frac{k-1}{6}|A_2|$. Since $\frac{2k+4}{3} > \frac{k-1}{6}$, we have $(k - k_2 - r_2)|A_2| > k'_2|A_2| = k_1|A_1|$. Then $|\omega^+(A)| = |A_1|(k - k_1 - r_1) + |A_2|(k - k_2 - r_2) > |A_1|(k - k_1 - r_1) + |A_1|k_1 = |\omega^+(A_1)|$ contradicts the fact that A is a λ-*atom*. Thus only the vertices in one orbit of A have out-neighbors in $V\backslash A$. Analogously, we can prove that only the vertices in one orbit of A have in-neighbors in $V\backslash A$. The proof is complete. □

A k-regular digraph D with girth g is called a (k, g)-digraph, and $n(k, g)$ is the minimum number of vertices a (k, g)-digraph can possess. Let D be the digraph with vertex set $V = \{0, 1, 2, \cdots, k(g - 1)\}$ and arc set $E = \{(i, j) \in V \times V \,|\, j - i \equiv m(mod(k(g-1)+1))\, and\, 1 \leqslant m \leqslant k\}$. Clearly, D is a vertex transitive (k, g)-digraph and, therefore, $n(k, g) \leqslant k(g - 1) + 1$ [1]. Hamidoune [3] proved that the girth of vertex transitive digraph of order n and regular degree k is less or equal to $\lceil n/k \rceil$, which implies $n \geqslant k(g-1)+1$. Thus, we have the following:

Lemma 2.8. *If D is vertex transitive (k, g)-digraph, then $|V(D)| \geqslant n(k, g)$ $= k(g - 1) + 1$.*

Clearly, if the degree of every vertex of vertex transitive digraph D is at least k, and the girth of D is at least g, then $|V(D)| \geqslant n(k, g)$. If D is a k-regular connected digraph with a directed cycle of length g_0 and $|V(D)| < n(k, g_0)$, then $g(D) < g_0$.

Now we give our main result:

Theorem 2.9. *If D is a k-regular connected digraph with two orbits and girth $g(D) \geqslant k$, then $\lambda(D) = k$.*

Proof. By contradiction, suppose $\lambda(D) < k$. Use notation as the above. By Lemma 2.7, without loss of generality, we assume that only the vertices in A_1 have out-neighbors in $V\backslash A$, then $k = k_2 + r_2$, $k - k_1 - r_1 \geqslant 1$ and $\lambda = |A_1|(k - k_1 - r_1) < k$. Thus $|A_1| \leqslant \lambda < k$. Since $g(D) \geqslant k$, we have $g(D[A_i]) \geqslant k$, this implies that $|A_i| \geqslant k$ if $r_i \geqslant 1$. Thus if $|A_i| < k$, we have $r_i = 0$ $(i = 1, 2)$. By Lemma 2.7, we consider two cases:

Case 1. Only the vertices in A_1 have in-neighbors in $V\backslash A$. Combining this with only the vertices in A_1 have out-neighbors in $V\backslash A$, we have the vertices in A_2 are not adjacent to the vertices in $V\backslash A$. Thus, we have $k_i = k'_i\,(i=1,2)$ and $k_1|\,A_1\,| = k_2|A_2|$. Since $\lambda = |A_1|(k-k_1-r_1) < k$, we have $|A_1| < k$, thus $r_1 = 0$. So $0 < \lambda = |A_1|(k-k_1) < k$ and $k - k_1 \geqslant 1$.

Subcase 1.1. $|A_1| < \lambda$. Since $\lambda = |A_1|(k-k_1)$ and $|A_1| < \lambda$, we have $k - k_1 \geqslant 2$, so $k_1 \leqslant k-2$. Then $k > \lambda = |A_1|(k-k_1) \geqslant 2|A_1|$. This gives that $|A_1| \leqslant \frac{k-1}{2}$, thus $k_2 \leqslant \frac{k-1}{2}$.

Subcase 1.1.1. $k_2 = 1$. Since $k_2 + r_2 = k$, we have $r_2 = k-1$, that is $D[A_2]$ is $(k-1)$-regular and vertex transitive. By Lemma 2.8, we have $|A_2| = v(D[A_2]) = k_1|A_1| < (k-1)\frac{k-1}{2} < (k-1)^2 + 1 = n(k-1,k)$. This implies that $g(D[A_2]) < k$, so $g(D) < k$, contradicting $g(D) \geqslant k$.

Subcase 1.1.2. $k_2 \geqslant 2$. Since $k_2 \leqslant \frac{k-1}{2}$, we have $r_2 = k - k_2 \geqslant \frac{k+1}{2}$. By Lemma 2.8, we have $|A_2| = v(D[A_2]) \geqslant n(\frac{k+1}{2},k) = \frac{k+1}{2}(k-1)+1 = \frac{k^2+1}{2}$. On the other hand, since $k_2|A_2| = k_1|A_1|$, $|A_1| \leqslant \frac{k-1}{2}$, $k_2 \geqslant 2$ and $k_1 < k-1$, we have $|A_2| = \frac{k_1}{k_2}|A_1| \leqslant \frac{k_1}{2k_2}(k-1) < \frac{(k-1)^2}{4} < \frac{k+1}{2}\cdot(k-1)+1 = \frac{k^2+1}{2}$, that is $|A_2| < \frac{k^2+1}{2}$, a contradiction.

Subcase 1.2. $|A_1| = \lambda$. Since $\lambda = |A_1|(k-k_1)$ and $|A_1| = \lambda < k$, we have $k_1 = k-1$ and $|A_1| \leqslant k-1$.

Subcase 1.2.1. $k_2 = 1$. Since $k_2 = 1$ and $k_2 + r_2 = k$, we have $r_2 = k-1$, this implies that $D[A_2]$ is $(k-1)$-regular and vertex transitive. Thus, $|A_2| = v(D[A_2]) = k_1|A_1| \leqslant (k-1)^2 < (k-1)^2+1 = n(k-1,k)$. By Lemma 2.8, we have $g(D[A_2]) < k$, this implies that $g(D) < k$, contradicting $g(D) \geqslant k$.

Subcase 1.2.2. $k_2 \geqslant 2$. Since $k_1 = k-1$ and $|A_1| \leqslant k-1$, we have $k_2|\,A_2\,| = k_1|A_1| \leqslant (k-1)^2$, and thus $|A_2| \leqslant \frac{1}{k_2}(k-1)^2$. Since $k_2 \leqslant |A_1| \leqslant k-1$ and $k_2 + r_2 = k$, we have $r_2 \geqslant 1$, this implies that $|A_2| \geqslant k$. Since $k \leqslant |A_2| \leqslant \frac{k-1}{k_2}\cdot(k-1)$, that is $k \leqslant \frac{k-1}{k_2}\cdot(k-1)$, we have $k-1 > k_2$ and so $k_2 \leqslant k-2$, combining this with $k_2 + r_2 = k$, we have $r_2 \geqslant 2$. By Lemma 2.8, we have $|A_2| \geqslant n(2,k) = 2(k-1)+1$. Since $|A_2| \leqslant \frac{1}{k_2}(k-1)^2$, we have $2(k-1)+1 \leqslant \frac{k-1}{k_2}\cdot(k-1)$. Thus $\frac{k-1}{k_2} > 2$, that is $k_2 < \frac{k-1}{2}$. Since $k_2 + r_2 = k$, we have $r_2 > \frac{k+1}{2}$. By Lemma 2.8, we have $|A_2| \geqslant n(\lceil\frac{k+1}{2}\rceil,k) \geqslant \frac{k+1}{2}\cdot(k-1)+1 > \frac{(k-1)^2}{2}+1$. But since $|A_2| \leqslant \frac{(k-1)^2}{k_2}$ and $k_2 \geqslant 2$, we have $|A_2| < \frac{(k-1)^2}{2}+1$, a contradiction.

Case 2. Only the vertices in A_2 have in-neighbors in $V\backslash A$. Combining this with only the vertices in A_1 have out-neighbors in $V\backslash A$, we have $k_2 + r_2 = k, k'_1 + r_1 = k$, and thus $0 < \lambda = |A_1|(k-k_1-r_1) = |A_2|(k-k'_2-r_2) < k$. Clearly, both $D[A_1]$ and $D[A_2]$ contain less than k vertices. Since $g(D) \geqslant k$, we have $r_1 = r_2 = 0$, so $k_2 = k$. But $k_2 \leqslant |A_1| < k$, that is $k_2 < k$, a contradiction.

In all cases we obtain contradictions, thus $\lambda(D) = k$. □

Now we give an example to show that the girth bound k is best possible.

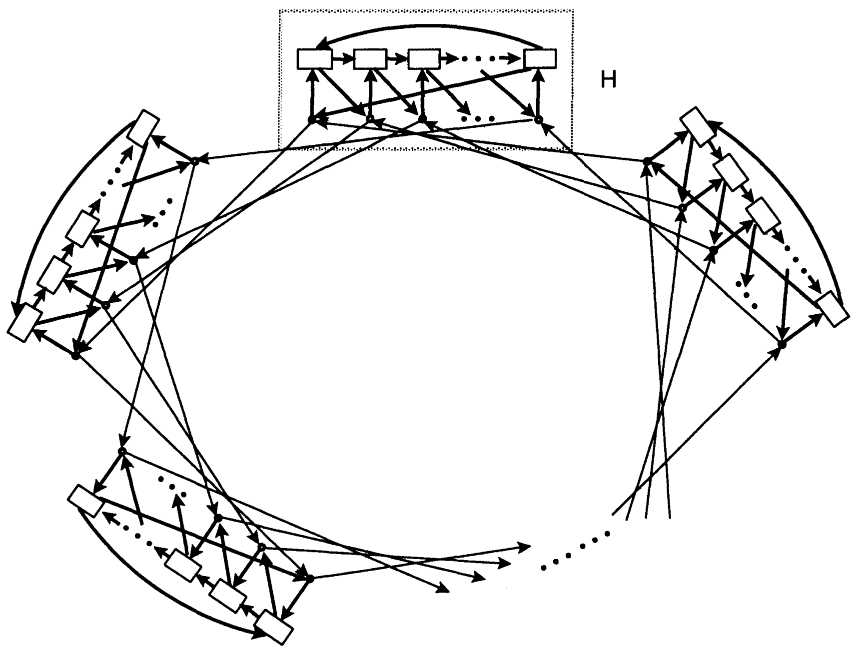

Figure 1

 is an independent set with k−1 vertices

 from A to B is an arc set with arcs from each vertex of A to each vertex of B

Example 2.10. *Let H be a strongly connected digraph with vertex set* $\{v_1, v_2, \cdots, v_{k-1}\}$ *and* $\{v_{i,1}, v_{i,2}, \cdots, v_{i,k-1}\}$ $(i = 1, 2, \cdots, k-1)$, *arc set*

$\{(v_{j,j_1}, v_{j+1,j_2}) \,|\, v_{j,j_1}, v_{j+1,j_2} \in V(H)\}$ $(j_1, j_2 = 1, 2, \cdots, k-1; j = 1, 2, \cdots, k-2)$;

$\{(v_{k-1,j_1}, v_{1,j_2}) \,|\, v_{k-1,j_1}, v_{1,j_2} \in V(H)\}$ $(j_1, j_2 = 1, 2, \cdots, k-1)$;

$\{(v_j, v_{j,j_1}) \,|\, v_j, v_{j,j_1} \in V(H)\}$ $(j, j_1 = 1, 2, \cdots, k-1)$;

$\{(v_{j,j_1}, v_{j+1}) \,|\, v_{j,j_1}, v_{j+1} \in V(H)\}$ $(j_1 = 1, 2, \cdots, k-1; j = 1, 2, \cdots, k-2)$;

$\{(v_{k-1,j_1}, v_1) \,|\, v_{k-1,j_1}, v_1 \in V(H)\}$ $(j_1 = 1, 2, \cdots, k-1)$.

The digraph $D_{k-1}(H)$ (see Figure. 1) is constructed by taking $k-1$ copies of H, $H_1, H_2, \cdots, H_{k-1}$, and adding arcs

$\{(v_j^i, v_j^{i+1}) \,|\, v_j^i \in V(H_i), v_j^{i+1} \in V(H_{i+1})\}$ $(i = 1, 2, \cdots, k-2; j = 1, 2, \cdots, k-1)$;

$\{(v_j^{k-1}, v_j^1) \,|\, v_j^{k-1} \in V(H_{k-1}), v_j^1 \in v(H_1)\}$ $(j = 1, 2, \cdots, k-1)$.

Clearly, $D_{k-1}(H)$ is the digraph with two orbits and girth $g = k-1$, but not maximally arc connected.

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